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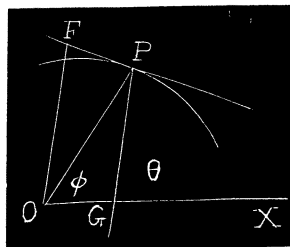
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## THE LENGTH OF A DEGREE OF LATITUDE AND LONGITUDE FOR ANY PLACE.

By **GEORGE B. McCLELLAN ZERR, A. M.**, Ph. D., Professor of Chemistry and Physics, The Temple College,  
Philadelphia, Pa.

Let  $O$  be the center of the earth,  $P$  any place on the surface,  $PG$  the normal and  $PF$  the tangent at  $P$ ,  $OP=p$ =the perpendicular from the center on the tangent.  $a, b$  the semi-axes of the earth,  $\rho$ =radius of curvature at  $P$ ,  $OP=r$ =the radius vector,  $\angle POX=\phi$ ,  $\angle PGX=FOX=\theta$ ,  $l$ =length of a degree of meridian,  $\rho'$ =radius of circle of latitude,  $L$ =length of degree of latitude,  $e^2=\frac{a^2-b^2}{a^2}=00680349$ =square of the eccentricity, since  $a=6378190$  meters=3963.296 miles.



$$\text{Now } \rho = \frac{a^2 b^2}{p^3}, \text{ but } p = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} = a \sqrt{1 - e^2 \sin^2 \theta}.$$

$$\therefore \rho = \frac{a(1-e^2)}{(1-e^2 \sin^2 \theta)^{\frac{3}{2}}}, \quad l = \frac{2\pi\rho}{360} = \frac{\pi\rho}{180}.$$

$$\therefore l = \frac{\pi a(1-e^2)}{180(1-e^2 \sin^2 \theta)^{\frac{3}{2}}} = \frac{68.70175}{(1-e^2 \sin^2 \theta)^{\frac{3}{2}}} \text{ miles} = \frac{110562.7346}{(1-e^2 \sin^2 \theta)^{\frac{3}{2}}} \text{ meters.}$$

$$\frac{1}{(1-e^2 \sin^2 \theta)^{\frac{3}{2}}} = 1 + \frac{3}{2}e^2 \sin^2 \theta + \frac{15}{8}e^4 \sin^4 \theta + \frac{105}{48}e^6 \sin^6 \theta + \dots$$

The fourth term can be omitted as its greatest value will not affect the result more than three inches.

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta), \quad \sin^4 \theta = \frac{1}{8}(3 - 4\cos 2\theta + \cos 4\theta).$$

These values, with the value of  $e$ , give

$$\frac{1}{(1-e^2 \sin^2 \theta)^{\frac{3}{2}}} = 1.00514058 - .00515323 \cos 2\theta + .00001265 \cos 4\theta.$$

$$\therefore l = 69.054917 - .354036 \cos 2\theta + .000869 \cos 4\theta \text{ miles}$$

$$= 111131.09118 - 569.75520 \cos 2\theta + 1.39862 \cos 4\theta \text{ meters.}$$

$$\rho' = r \cos \phi = \frac{a \cos \theta}{\sqrt{1-e^2 \sin^2 \theta}}.$$

$$L = \frac{2\pi\rho'}{360} = \frac{\pi\rho'}{180} = \frac{\pi a \cos\theta}{180\sqrt{(1-e^2\sin^2\theta)}}.$$

$$\therefore L = \frac{69.1726\cos\theta}{\sqrt{(1-e^2\sin^2\theta)}} \text{ miles} = \frac{111320.4635\cos\theta}{\sqrt{(1-e^2\sin^2\theta)}} \text{ meters.}$$

$$\frac{1}{\sqrt{(1-e^2\sin^2\theta)}} = 1 + \frac{1}{2}e^2\sin^2\theta + \frac{3}{8}e^4\sin^4\theta + \frac{15}{48}e^6\sin^6\theta + \dots$$

$$= (1 + \frac{1}{2}e^2 + \frac{3}{8}e^4) - (\frac{1}{2}e^2 + \frac{3}{4}e^4)\cos^2\theta + \frac{3}{8}e^4\cos^4\theta \dots$$

$$\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta), \quad \cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta).$$

$$\therefore L = 111320.4635[(1 + \frac{1}{2}e^2 + \frac{3}{8}e^4)\cos\theta - (\frac{1}{2}e^2 + \frac{9}{16}e^4)\cos 3\theta + \frac{3}{16}e^4\cos 5\theta]$$

$$= 111415.37533\cos\theta - 95.03428\cos 3\theta + .12022\cos 5\theta.$$

Since the greatest value of the last term is not over five inches it can be omitted.

$$\therefore L = 111415.37533\cos\theta - 95.03428\cos 3\theta \text{ meters}$$

$$= 69.23155\cos\theta - .05905\cos 3\theta \text{ miles.}$$

The following table gives the length of a degree at intervals of five degrees.

<i>Degrees</i>	$\overbrace{\hspace{1.5cm}}^L$		$\overbrace{\hspace{1.5cm}}^L$	
	<i>Meters</i>	<i>Miles</i>	<i>Meters</i>	<i>Miles</i>
0°	110562.7346	68.70175	111320.3411	69.17250
5°	110571.305	68.70701	111051.725	69.00558
10°	110596.769	68.72288	109640.673	68.12878
15°	110638.365	68.74873	107552.254	66.83108
20°	110694.879	69.78386	104648.397	65.02667
25°	110764.615	68.82719	100952.272	62.72996
30°	110845.514	68.87746	96489.058	59.95660
35°	110935.152	68.93315	91290.501	56.72631
40°	111030.839	68.99261	85396.151	53.06366
45°	111129.693	69.05405	78850.126	48.99608
50°	111228.715	69.11556	71698.992	44.55249
55°	111324.887	69.17532	63997.427	39.96687
60°	111415.270	69.23150	55802.722	34.67483
65°	111497.081	69.28234	47178.162	29.31568
70°	111567.789	69.32627	38188.589	23.72972
75°	111625.216	69.36194	28903.727	17.96027
80°	111667.556	69.38825	19394.797	12.05159
85°	111693.506	69.40438	9735.561	6.04951
90°	111702.245	69.40988	0.000	0.000